This practical combines written tasks and an opportunity to explore the models using numerical examples in Matlab. Matlab tasks are given in boxes and require you to use the ODE_Interface.

**Question 1: Predator-prey dynamics**

(i) Write down the pair of differential equations describing the simple Lotka-Volterra predation model for the interaction of a predator $S$ and its prey $F$, in the absence of intra-specific density-dependent constraints on the two species. Give a brief biological interpretation of the model's parameters.

(ii) Calculate the positive equilibrium point and plot null-clines for the model.

**Matlab Task 1**
Choose positive parameter values which give an equilibrium point at (5,4) and use Matlab to investigate the stability of the equilibrium points.

(iii) Now re-write the model, adding logistic intra-specific density dependence (with carrying capacity $K$) to the prey growth rate. Again, calculate and plot null-clines for the model, identifying the point where both populations have positive equilibria.

(iv) For the new model, examine with a local stability analysis whether the positive equilibrium for both species is locally stable.

**Matlab Task 2**
Choose positive parameter values for the modified model which give an equilibrium point at (5,4) and use Matlab to investigate the stability of the equilibrium points.

(v) A change in climate `enriches' the prey population, doubling its carrying capacity, $K$. Briefly comment, **without additional algebraic details**:

(a) whether this change will increase the prey equilibrium and  
(b) how this change affects the predator population (**hint**: consider how increasing $K$ alters the prey nullcline).

**Matlab Task 3**
Using the same parameter values and model as for Task 2 investigate the effect of varying the value of the carrying capacity K on the predicted population dynamics.
Question 2: Competition

Two insect species, \( N_1 \) and \( N_2 \) interact according to the following pair of differential equations

\[
\frac{dN_1}{dt} = r_1 N_1 \left( 1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right) \\
\frac{dN_2}{dt} = r_2 N_2 \left( 1 - \frac{N_2 + \alpha_{21} N_1}{K_2} \right)
\]

(i) Briefly, give a biological interpretation of the model’s parameters. What interspecific interaction does the model reflect?

(ii) Derive expressions for equilibria of the model. Assuming that the equilibria for both species are positive, use a local stability analysis to derive an expression (involving \( \alpha_{12} \) and \( \alpha_{21} \) which needs to be satisfied for these equilibria to be locally stable.

(iii) Field studies show that, if the two species are introduced together into an area, only one persists - either \( N_1 \) or \( N_2 \) - depending on the relative numbers of the two species introduced. Assuming that the population parameters (\( r_1, k_1, \alpha_{12} \) etc) are always the same and there is a positive equilibrium for the two species, draw nullcline diagrams to illustrate how the exclusion of either species by the other depending on initial conditions could occur.

(iv) Comment briefly on whether the equilibrium in (iii) when one species or other is excluded is locally stable or unstable.

Matlab Task 4
Verify your results in Matlab initially setting \( r_1 = 0.1, r_2 = 0.5, K_1 = 5, K_2 = 4, \alpha_{12} = 0.3, \alpha_{21} = 0.5 \) then explore the effect of varying the parameters using the results of the stability analysis above to help you obtain (a) a positive equilibrium which is a saddle point, (b) no positive equilibrium value.

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